MINISTRY OF EDUCATION

M 101

GIMNÁZIUM
May 2005

Solutions and Markscheme in Physics

I. Numerical Problems

1. (10.14.)

A stone is thrown vertically downwards with an initial speed of 5 m/s. How long does it take the kinetic energy of the stone to increase by a factor of four? (g = 10 m/s²)

Data: $v_o = 5$ m/s, $E / E_o = 4$, $g = 10$ m/s^2.

The final speed of the stone can be calculated from the given information on the change in kinetic energy:

$$\frac{E}{E_o} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv_o^2} = \frac{v^2}{v_o^2} = 4 \quad \Rightarrow \quad v = 2v_o = 10\,\frac{\text{m}}{\text{s}}.$$

5 marks

The time t of the motion can be obtained by applying the laws of kinematics to projection vertically downwards:

$$v = v_o + gt \quad \Rightarrow \quad t = \frac{v - v_o}{g} = 0.5\ \text{s}.$$

It takes 0.5 seconds for the kinetic energy of the stone to increase by a factor of four.

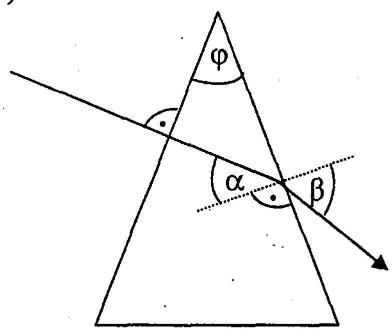
5 marks

Subtotal: 10 marks

2. (23.26.)

We want a light ray incident normally on one face of a prism to leave through the other face with a 45° angle of refraction. What should be the size of the vertex angle, given that the refractive index of the material of the prism is 1.5 ?

(The vertex angle of the prism is the angle enclosed by its two refracting faces.)



Data: $\beta = 45°$, $n = 1.5$.

The path of the light ray is shown in the accompanying diagram.

3 marks

The angle of incidence α on the second refracting face can be calculated from the law of refraction:

$$\frac{\sin(\alpha)}{\sin(\beta)} = \frac{1}{n} \quad \Rightarrow \quad \sin(\alpha) = \frac{\sin(\beta)}{n} \quad \Rightarrow \quad \alpha = 28.13°$$

7 marks

Since the arms of the angle of incidence α and the vertex angle φ of the prism are pairwise perpendicular, the two angles are equal:

$\varphi = \alpha = 28.13°$. The vertex angle of the prism should be 28.13°.

5 marks

Subtotal: 15 marks

3. (26.29.)

2 m³ of nitrogen gas has a temperature of 77 °C. It absorbs $3.5 \cdot 10^6$ J of heat at a constant pressure of $3 \cdot 10^5$ Pa.
a) Find the final volume and temperature of the gas.
b) Find the change in the energy of the gas owing to the absorption of heat.
c) Find the work done by the gas while being heated.
(The value of the Boltzmann constant is $k = 1.38 \cdot 10^{-23}$ J/K.)

Data: $p = 3 \cdot 10^5$ Pa, $V_1 = 2$ m³, $T_1 = 77$ °C $= 350$ K, $Q = 3.5 \cdot 10^6$ J.

a) It is known that the heat absorbed during isobaric heating can be expressed in terms of the state variables as follows:

$$Q = \frac{f+2}{2} p\Delta V$$

Using that $f = 5$ for nitrogen gas, the change in volume can be calculated:

$$\Delta V = \frac{2Q}{(f+2)p} = 3.33\ m^3 .$$

The volume of the gas after the heat absorption is $V_2 = V_1 + \Delta V = 5.33\ m^3$.

4 marks

The temperature T_2 of the gas after heating is obtained by using the equation of state for the ideal gas:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad \Rightarrow \quad T_2 = \frac{V_2}{V_1} T_1 = 933.3\ \text{K} = 660.3\text{°C}.$$

4 marks

b) The change in the energy of the gas can be calculated from the known state variables:

$$\Delta E = \frac{f}{2} p\Delta V = 2.50 \cdot 10^6\ \text{J} .$$

4 marks

c) The work done by the gas is

$$W = Q - \Delta E = p\Delta V = 1.00 \cdot 10^6\ \text{J} .$$

3 marks

Subtotal: 15 marks

4. (16.23.)

An electron accelerated from rest through a potential difference of 1500 V enters a uniform magnetic field in a direction perpendicular to the magnetic induction vector.

a) To what speed is the electron accelerated in the electric field?

b) Find the magnitude of the magnetic induction vector, given that the electron travels in a circle of radius 1 cm in the magnetic field.

(The magnitude of the charge of the electron is $1.6 \cdot 10^{-19}$ C, its mass is $9.1 \cdot 10^{-31}$ kg.)

Data: $U = 1500$ V, $R = 1$ cm, $e = 1.6 \cdot 10^{-19}$ C, $m = 9.1 \cdot 10^{-31}$ kg.

a) The speed of the accelerated electron can be obtained by using the work–energy theorem:

$$\frac{1}{2} mv^2 = eU \quad \Rightarrow \quad v = \sqrt{\frac{2eU}{m}} = 2.30 \cdot 10^7\ \frac{m}{s} .$$

8 marks

b) In the magnetic field, the electron is moving uniformly in a circle. Applying the law of dynamics, we have:

$$m\frac{v^2}{R} = evB \quad \Rightarrow \quad B = \frac{mv}{eR} = 1.31\cdot 10^{-2}\ T.$$

12 marks

Subtotal: 20 marks

II. Analysis of an Experiment (15.68.)

5. A variable resistor was connected to an accumulator, and the currents corresponding to the given values of the resistance were measured. The data obtained are shown in the table below.

R (Ω)	2	4	6	8	10	12	14	16	18	20
I (mA)	83.3	71.0	62.2	55.6	50.1	45.4	41.6	38.4	35.7	33.3

a) Using the data, determine the internal resistance and electromotive force of the accumulator.
b) For the values of the variable resistance given in the table, find the electric power appearing on the resistor, and then plot the values obtained in a power versus resistance graph. Use the graph to determine the resistance for which the power drawn by the resistor from the circuit is a maximum.

Solution:

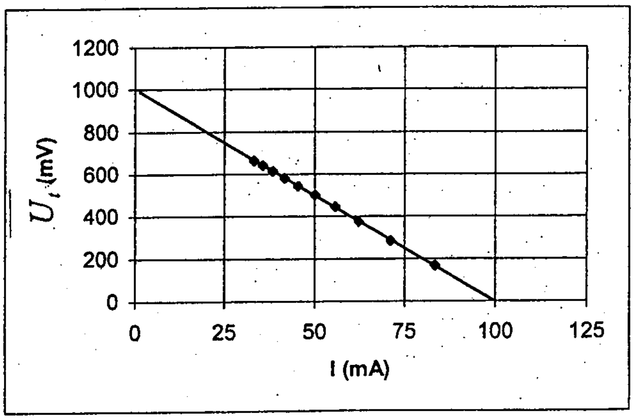
a) The terminal voltages ($U_t = R\,I$) can be determined from the measured data.

R (Ω)	2	4	6	8	10	12	14	16	18	20
I (mA)	83.3	71	62.2	55.6	50.1	45.4	41.6	38.4	35.7	33.3
U_t (mV)	166.6	284	373.2	444.8	501	544.8	582.4	614.4	642.6	666

If a battery of electromotive force ε and internal resistance R_{int} supplies a current I then the terminal voltage U_t satisfies

$$U_t = \varepsilon - R_{\text{int}} I\ .$$

The function $U_t(I)$ is linear and the intercepts of its graph on the axes are the electromotive force ε and the short circuit current $I_{max} = \varepsilon / R_{int}$.



If the measured values of U_t are graphed against I and the best fitting line is drawn, the electromotive force and short circuit current of the battery can be read from the graph:

$$\varepsilon = 1000\text{ mV} = 1\text{ V},\quad I_{max} = 100\text{ mA}.$$

The internal resistance of the battery is $$R_{\text{int}} = \frac{\varepsilon}{I_{\text{max}}} = 10\ \Omega.$$

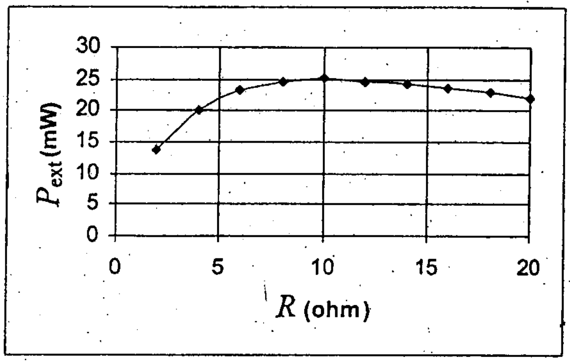
12 marks

b) The power that appears on the external resistance can be calculated for each resistance R applied ($P_{ext} = R\,I^2$).

R (Ω)	2	4	6	8	10	12	14	16	18	20
I (mA)	83.3	71	62.2	55.6	50.1	45.4	41.6	38.4	35.7	33.3
P_{ext} (mW)	13.88	20.16	23.21	24.73	25.10	24.73	24.23	23.59	22.94	22.18

The P_{ext} versus R graph shows that the power is a maximum for $R = 10\ \Omega$. (In accordance with the known fact that the external power is a maximum if the external and internal resistances are equal.)

8 marks



Subtotal: 20 marks

<u>III. Theoretical Question</u> (3.41.)

6. State the law of momentum conservation. (Your response should contain the definition of momentum, the statement of the law, a description of the conditions for the law to be applicable, and the description of at least two particular phenomena where the law is valid.)

The momentum of a particle of mass m and velocity $\vec{v}$ is <u>defined</u> as the product of the mass and velocity. The most common notations for momentum are $\vec{p}$ or $\vec{I}$, thus $\vec{p} = m\vec{v}$. Momentum is a vector quantity, its direction is the same as that of the velocity vector. The unit of momentum is the kg·m/s.

6 marks

(In the case of a system of particles, the momentum of the system is defined as the sum of the momenta of the individual particles. The momentum of a body or set of bodies having a continuous mass distribution can be defined in a similar way. It can be shown that the momentum of a system of particles, a body or set of bodies can also be obtained as the product of the total mass and the velocity of the centre of mass.)

<u>The law of momentum conservation</u>: In an isolated system, the sum of the momenta is constant. In the case of two objects, for example, $m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{u}_1 + m_2\vec{u}_2$ with conventional notations.

6 marks

A system of objects is said to be <u>isolated</u> if either the objects constituting the system are not acted on by any external forces at all or the resultant of all external forces is zero. (An external force is one exerted by an object or field that does not belong to the system.)

4 marks

The candidate is expected to describe at least <u>two particular situations</u> in which the law of momentum conservation is applicable. (For example, collisions, recoil, explosions, the decay of an atomic nucleus.)

4 marks

Subtotal: 20 marks

EVALUATION

A total of 100 marks can be received on this paper.
From 0 to 19 inclusive: fail (1); 70 and above: excellent (5).
The teacher may depart from the lower boundary of the excellent grade (70 marks) and the upper boundary of the fail grade (19 marks) by at most ±3 marks.
The setting of the other grade boundaries is left to the teacher's professional judgment.
A correct solution different from the one given in the markscheme should receive full marks.
Evaluate only one solution of each problem.