Azonosító jel: | | | | | | | | | | | | | | |

ÉRETTSÉGI VIZSGA ● 2013. május 7.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

2013. május 7. 8:00

Az írásbeli vizsga időtartama: 240 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to candidates

1. The time allowed for this examination paper is 240 minutes. When that time is over, you will have to stop working.

2. You may solve the problems in any order.

3. In Section II, you are only required to solve four out of the five problems**. When you have finished the examination, write in the square below the number of the problem NOT selected.**
   If it is not clear to the examiner which problem you do not want to be assessed, then problem 9 will not be assessed.

4. In solving the problems, you are allowed to use a calculator that cannot store and display verbal information. You are also allowed to use any book of four-digit data tables. The use of any other electronic devices, or printed or written material is forbidden.

5. **Always write down the reasoning used in obtaining the answers, since a large part of the attainable points will be awarded for that.**

6. **Make sure that the calculations of intermediate results are also possible to follow.**

7. In solving the problems, theorems studied and given a name in class (e.g. the Pythagorean theorem or the altitude theorem) do not need to be stated precisely. It is enough to refer to them by the name, but their applicability needs to be briefly explained. Reference to other theorem(s) will only be awarded full mark if the theorem and all its conditions are stated correctly (proof is not required), and the applicability of the theorem to the given problem is explained.

8. Always state the final result (the answer to the question of the problem) in words, too.

9. Write in pen. The examiner is instructed not to mark anything in pencil, other than diagrams. Diagrams are also allowed to be drawn in pencil. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.

10. Only one solution to each problem will be assessed. In the case of more than one attempt to solve a problem**, indicate clearly** which attempt you wish to be marked.

11. Do not write anything in the grey rectangles.

I.

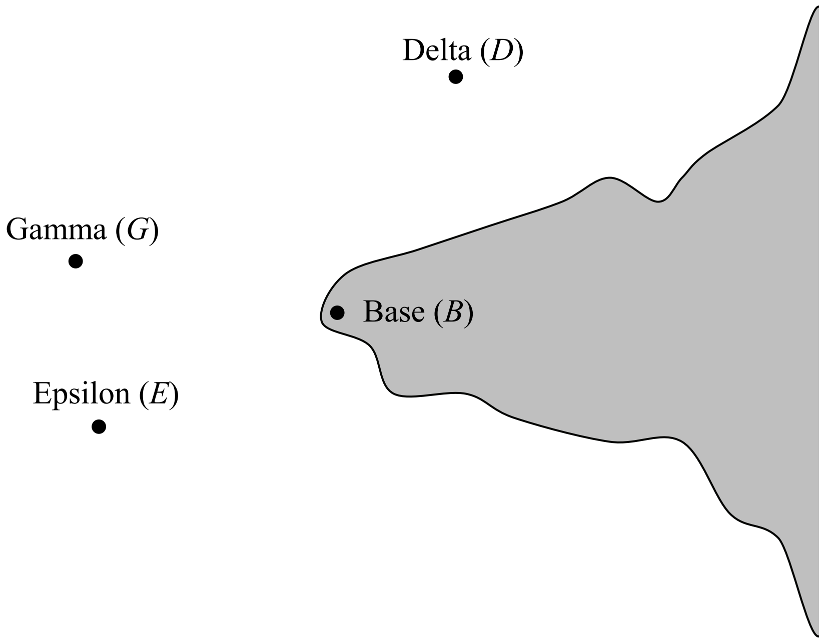
1. Solve the following inequalities on the set of real numbers:

a) $\log_{\frac{1}{5}}(2x-1) < 0$

b) $2^{|2x-1|-2} > 1$

a)	4 points	
b)	6 points	
T.:	10 points	

2. An oil company operates three marine oil wells (named Gamma, Delta and Epsilon) around a peninsula. The company base is situated on the peninsula. The positions of the oil wells and the base are shown in the map below, on a scale of 1 : 500 000. On the map, each oil well is at a distance of exactly 3.5 cm from the base, $\angle EBD = 142°$, and $\angle GED = 54°$.



a) Use the given data of the map to determine, in kilometres, the distance from the base to the wells.

On Mondays, a helicopter flies along the path Base–Epsilon–Gamma–Delta–Base to deliver food to the staff of the oil wells. On Thursdays, the same helicopter flies along the path Base–Gamma–Epsilon–Delta–Base, to refresh supplies.

b) Given that the helicopter always travels in straight lines, calculate the total distance the helicopter covers on Mondays, and the total distance it covers on Thursdays. Express your answer in kilometers, rounded to the nearest integer. (In your calculations, only consider the horizontal component of the helicopter's motion.)

a)	3 points	
b)	11 points	
T.:	14 points	

3.

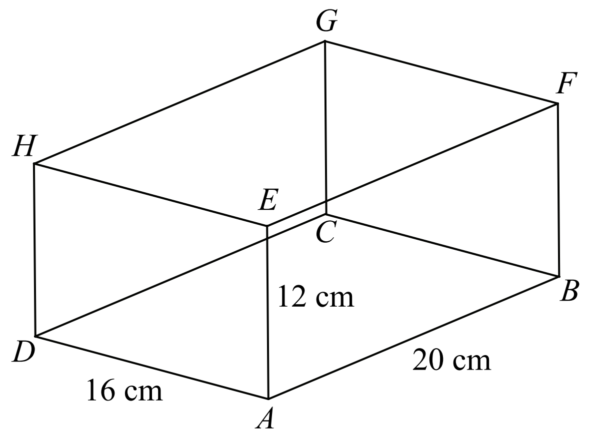
a) How many three-digit numbers of the form $\overline{abb}$ are there in base-three notation? (a and b do not necessarily denote different digits.)

List these numbers in base-three notation and in base-ten notation.
How many even numbers are there among them that consist of two digits in base-ten notation?

b) How many subsets of at least two elements does the set {2; 3; 4; 5; 6} have, such that the product of the elements of the subset is divisible by 3?

a)	5 points	
b)	8 points	
T.:	13 points	

4. The lengths of the edges from vertex *A* of the cuboid in the figure are *AB*=20 cm; *AD*=16 cm; *AE*=12 cm.



a) Let *P* denote the midpoint of edge *AB*, and let *Q* be the midpoint of edge *EH*. Calculate the distance *PQ*.

Two lines are selected in every possible way out of the lines of the edges of the cuboid.

b) How many different pairs of lines may be selected? (Two pairs of lines are considered different if at least one of the lines is different.)

c) How many of these pairs are intersecting pairs, how many are parallel pairs, and how many are pairs of skew lines?

d) Find the distances of line *AE* from those lines that are skew relative to line *AE*.

a)	4 points	
b)	3 points	
c)	4 points	
d)	3 points	
T.:	14 points	

II.

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

5.

a) The first term of a geometric progression is 32, and its common ratio is $\frac{1}{128}$.

Prove that no matter how many terms of the sequence are added starting with the first term, the sum cannot exceed 32.5 .

b) $\{a_n\}$ is a geometric progression in which the first term is $\frac{1}{128}$ and the common ratio is 32.

What may be the positive integer n if the equality $a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_n = 2048^{3n}$ holds?

a)	4 points	
b)	12 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

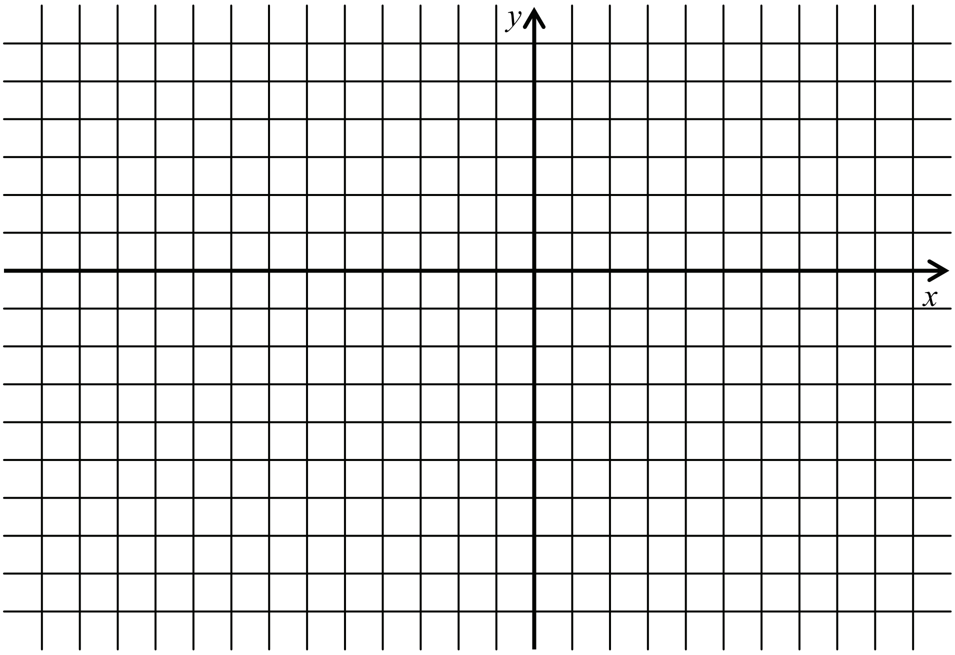
6. p is a real parameter, such that the parabolas given by the equations $y = x^2 + px + 1$ and $y = x^2 - x - p$ are different, and they have a common point on the x-axis.

a) Calculate the value of p, and write down the equations of the parabolas with this value of p.

Graph the parabolas given by the equations $y = x^2 + 2x$ and $y = x^2 - x - 3$ on the same set of coordinate axes.

b) Calculate the area of the region bounded by the two parabolas and the y-axis.

a)	8 points	
b)	8 points	
T.:	16 points	


y
x

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

7. Statistics made over several years by a cell phone company revealed that on average, one out of sixty text messages (SMS) sent off correctly did not get delivered to the addressee. The questions below refer to text messages transmitted by this cell phone company.

a) For each of the statements below, decide whether it is true or false. Indicate your answer by putting an × in the appropriate field of the table. (No explanation is required.)

	Statement	TRUE	FALSE
1.	If we send 45 text messages during a month, then it is certain that all of them will get delivered.		
2.	If we send off every SMS twice, then we can be sure that at least one of each pair will get delivered.		
3.	It is possible that out of the 5 text messages sent yesterday, only one got delivered to the addressee.		
4.	If we send 120 text messages in the next ten days, it is possible that all of them will be delivered.		
5.	If we sent 180 text messages during two days, then it is certain that three of them did not get delivered.		

In the considerations below, assume that the number of successfully sent text messages is binomially distributed.

b) If three text messages are sent, what is the probability that exactly one of them will not be delivered?

If you use rounded values in your calculations, then use values rounded to 4 decimal places.

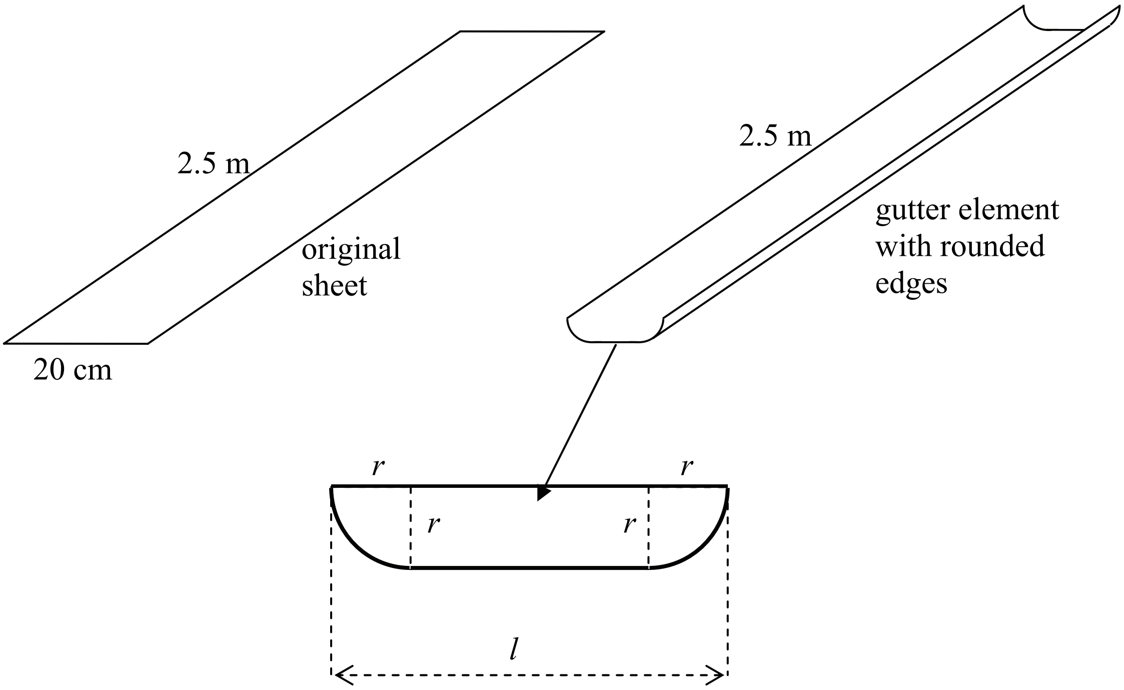
c) What is the minimum number of text messages that need to be sent so that the probability of at least one of them not being delivered is at least 98%?

If you use rounded values in your calculations, then use values rounded to 4 decimal places.

a)	5 points	
b)	4 points	
c)	7 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

8. In a tin shop, roof gutter elements of length 2.5 m are made out of thin rectangular sheets of tin. The sheets are 20 cm wide and 2.5 m long. The channel has a cross section with rounded edges, as shown in the figure.



a) The cross-sectional area of the channel, shown bounded by the solid line is 55 cm^2. What is the radius (r) of the quarter circles, and what is the width (l) of the channel? Express your answers in centimetres, rounded to one decimal place.

b) The designers wish to make a gutter element of maximum capacity. Prove that this is achieved if $l = 2r$. Calculate the amount of water, in litres, that a gutter element made with such a cross section can hold if it is in a horizontal position. (Round your answer to the nearest whole litre.)

a)	6 points	
b)	10 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

9. András is the most successful member of the basketball team of the school. The school team took part in a tournament of ten rounds. In the sixth, seventh, eighth and ninth rounds, András scored 23, 14, 11 and 20 points, respectively. After the ninth round, his points average was larger than his points average after the fifth round. At the end of the tournament, it turned out that his average score in the ten games was at least 18 points per game.
What is the minimum number of points that András may have scored in the tenth round?

T.:	16 points	

	number of problem	maximum score	points awarded	maximum score	points awarded
Section I	1.	10		**51**	
	2.	14			
	3.	13			
	4.	14			
Section II		16		**64**	
		16			
		16			
		16			
		← problem not selected			
Total score on written examination				**115**	

date

examiner

	score rounded to **integer** / elért pontszám **egész számra** kerekítve	**integer** score entered in program/ programba beírt **egész** pontszám
Section I / I. rész		
Section II / II. rész		

examiner / javító tanár

registrar / jegyző

date / dátum

date / dátum