ÉRETTSÉGI VIZSGA ● 2013. május 7.

# MATEMATIKA ANGOL NYELVEN

## KÖZÉPSZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

## JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

# Instructions to examiners

**Formal requirements:**

1. Mark the paper in **ink, different in colour** from the one used by the candidate. Indicate the errors, incomplete solutions, etc. in the conventional way.
2. The first one of the rectangles under each problem shows the maximum attainable score on that problem. The **points** given by the examiner are to be entered **in the rectangle** next to that.
3. **If the solution is perfect,** it is enough to enter the maximum scores in the appropriate rectangles.
4. If the solution is incomplete or incorrect, please indicate the individual **partial scores** in the body of the paper, too.
5. Except diagrams, do not assess anything that is written in pencil.

**Assessment of content:**

1. The markscheme may contain more than one solution for some of the problems. If the **solution by the candidate is different**, allocate the points by identifying the parts of the solution equivalent to those of the one given in the markscheme.
2. The subtotals in the markscheme can be **further divided**, but the scores awarded should always be whole numbers.
3. If it is clear that the reasoning and the final answer are both correct, you may award the maximum score even if the solution is **less detailed** than the one in the markscheme.
4. If there is a **calculation error** or inaccuracy in the solution, only take off the points for that part where the error occurs. If the reasoning remains correct and the error is carried forward, the points for the rest of the solution should be awarded.
5. In the case of a **principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the markscheme.) However, if the wrong information obtained owing to the principal error is carried forward to the next section or in the next part of the problem and is used correctly, the maximum score is due for the next part, provided that the error has not changed the nature of the task to be completed.
6. Where the markscheme shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.
7. If there are more than one different approaches to a problem, **assess only the one indicated by the candidate**.
8. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
9. **Do not take off points** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **Assess only two out of the three problems in part B of Paper II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted in their total score. Should there be a solution to that problem, it does not need to be marked. However, if it is still not clear which problem the candidate does not want to be assessed, assume automatically that it is the last one in the question paper, and do not assess that problem.

# I.

| **1.** | | |
|---|---|---|
| The simplified fraction: $\frac{a-2b}{3}$. | 2 points | *The 2 points cannot be divided.* |
| **Total:** | **2 points** | |

| **2.** | | |
|---|---|---|
| A right circular cylinder is obtained.<br>Base radius is 5 cm, height is 12 cm. | 1 point | *The point is due if these ideas appear in a diagram.* |
| $V = 25\pi \cdot 12$ (cm$^3$). | 1 point | |
| The volume of the right circular cylinder is $300\pi$ cm$^3$. | 1 point | *The answer is also accepted in decimal form.* |
| **Total:** | **3 points** | |

| **3.** | | |
|---|---|---|
| The number of real roots is 1. | 2 points | *1 point if the answer is x = 5, no points if the answer is 5.* |
| **Total:** | **2 points** | |

| **4.** | | |
|---|---|---|
| $x_1 = 14$, $x_2 = -14$ | 2 points | *1 point for each correct answer.* |
| **Total:** | **2 points** | |

| **5.** | | |
|---|---|---|
| The position vector of the midpoint is $\vec{f} = \frac{\vec{a}+\vec{b}}{2}$. | 1 point | |
| Rearranged: $\vec{b} = 2\vec{f} - \vec{a}$. | 1 point | |
| **Total:** | **2 points** | |

| **6.** | | |
|---|---|---|
| The smallest positive angle is 30º. | 2 points | *1 point for stating that 750º = 2·360º+30º.* |
| **Total:** | **2 points** | |

| **7.** | | |
|---|---|---|
| $x^2 + 18x + 81 = (x+9)^2$ | 1 point | |
| The square of a number is a minimum when 0 is squared.<br>The smallest value of the function occurs at $x = -9$. | 1 point | |
| **Total:** | **2 points** | |

| **8.** | | |
|---|---|---|
| There are $2^4$=16 positive five-digit numbers. | 2 points | |
| **Total:** | **2 points** | |

| **9.** | | |
|---|---|---|
| Group I: 180 people,<br>Group II: 240 people,<br>Group III: 300 people. | 1 point each | |
| **Total:** | **3 points** | |

| **10.** | | |
|---|---|---|
| Rearranged: $2x-7y=0$. | 1 point | |
| A normal vector of a perpendicular line *e* is **n** ( 7 ; 2 ), | 1 point | |
| Thus the equation of line *e* is $7x + 2y = 33$. | 1 point | *The equations of the lines are accepted in any correct form.* |
| **Total:** | **3 points** | |

| **11.** | | |
|---|---|---|
| A: true;<br>B: false;<br>C: true;<br>D: true. | 1 point each | . |
| **Total:** | **4 points** | |

| **12.** | | |
|---|---|---|
| The terms of the sequence are $a_1=-1,\quad a_2=1,\quad a_3=0,\quad a_4=1,\quad a_5=1,\quad a_6=2$. | 2 points | |
| Therefore $S_6=4$. | 1 point | |
| **Total:** | **3 points** | |

# II. A

| 13. a) | | |
|---|---|---|
| The side of the square is $a$, the sides of the rectangles are $a$ and $\frac{a}{3}$. | 1 point | *The point is also due if the notations only appear in a diagram.* |
| The perimeter of one rectangle is $2a+\frac{2a}{3}=24$, | 2 points | |
| hence $a=9$ cm. | 1 point | |
| The area of the square is 81 cm$^2$. | 1 point | |
| **Total:** | **5 points** | |

| 13. b) Solution 1 | | |
|---|---|---|
| From the Pythagorean theorem: $13^2-12^2=x^2$ (or: 13, 12, 5 are a Pythagorean triple), | 1 point | |
| the other leg of the right-angled triangle (BP) is 5 cm. | 1 point | |
| The area $T$ of the right-angled triangle can be expressed in two ways: $T=\frac{a\cdot b}{2}=\frac{c\cdot m_c}{2}$, where $m_c$ is the height drawn to hypotenuse $c$). | 2 points | |
| Hence $a\cdot b=c\cdot m_c$, | 1 point | |
| that is, $m_c=\frac{a\cdot b}{c}=\frac{60}{13}$. | 1 point | |
| The height drawn to the hypotenuse is 4.6 cm long. | 1 point | |
| **Total:** | **7 points** | |

| 13. b) Solution 2 | | |
|---|---|---|
| From the Pythagorean theorem: $13^2-12^2=x^2$ (or: 13, 12, 5 are a Pythagorean triple), | 1 point | |
| the other leg of the right-angled triangle (BP) is 5 cm. | 1 point | |
| It is known that the leg is the geometric mean of its projection and the hypotenuse: $5=\sqrt{13\cdot p}$, | 2 points | |
| $p=\frac{25}{13}\approx 1.92$. | 1 point | |
| From the Pythagorean theorem the square of the height $m_c$ drawn to the hypotenuse is $m_c{}^2=5^2-\left(\frac{25}{13}\right)^2$ | 1 point | |
| The height drawn to the hypotenuse is 4.6 cm long. | 1 point | |
| **Total:** | **7 points** | |

| **13. b)** **Solution 3** | | |
|---|---|---|
| Let α denote the angle at vertex A in the right-angled triangle *ABP*, and let Q be the foot of the altitude drawn to the hypotenuse.<br>In the right-angled triangle *ABP*, $\cos\alpha = \frac{AB}{AP} = \frac{12}{13}$. | 2 points | |
| $\alpha \approx 22.62°$. | 1 point | *This point is also due if the value of* sin α *is calculated correctly later on.* |
| In the right-angled triangle *AQB*, $\sin\alpha = \frac{BQ}{AB} = \frac{BQ}{12}$. | 2 points | |
| $BQ = 12 \cdot sin\,\alpha \approx 12 \cdot 0.3846 \approx 4.6152$. | 1 point | |
| The height drawn to the hypotenuse is 4.6 cm long. | 1 point | |
| **Total:** | **7 points** | |

| **14. a)** | | |
|---|---|---|
| Domain of definition:<br>$(\text{since } 2x-5>0 \text{ and } x>0)$, $x > \frac{5}{2}$. | 1 point | *This point is also due if the candidate uses the equation obtained by cancelling the logarithms, but checks the root by substitution.* |
| (It follows from the identities of logarithms that)<br>$2x-5=\frac{x}{3}$. | 2 points | |
| Hence $x = 3$. | 1 point | |
| This root is in the domain of definition, therefore it is the solution of the equation. | 1 point | |
| **Total:** | **5 points** | |

| 14. b) | | |
|---|---|---|
| $0 \le 13-2x$, so $x \le 6{,}5$. | 1 point | *These points are also due if the candidate uses the equation obtained by squaring, checks the results by substitution and rejects the extraneous root.* |
| $0 \le \sqrt{13-2x} = x-5$, hence $5 \le x$. Thus the equation may only have a solution $5 \le x \le 6.5$. | 1 point | |
| If both sides are squared, the left-hand side will be $13-2x$. | 1 point | |
| The square of the right-hand side: $x^2-10x+25$. | 1 point | |
| The quadratic equation obtained: $0 = x^2-8x+12$. | 1 point | |
| Hence $x=6$ or $x=2$. | 1 point | |
| The only solution on the domain of the equation is 6. | 1 point | |
| **Total:** | **7 points** | |

| 15. a) | | |
|---|---|---|
| There are 20 employees who have both kinds of diploma, | 1 point | |
| since the total number of diplomas is 42+28=70, which is 20 more than the number of diploma holders. | 1 point | *Award this point for a correct set diagram, too.* |
| Thus there are 22 with technician's certificates only. | 1 point | |
| **Total:** | **3 points** | |

| 15. b) | | |
|---|---|---|
| If the number of those under 30 is *x*, | 1 point | *The point is also due if this idea is only reflected by the solution.* |
| then the mean is $\frac{x \cdot 148000 + (50-x) \cdot 173000}{50} = 165\,000.$ | 1 point | |
| $x = 16$ | 1 point | |
| There are 16 employees under 30 in the lab. | 1 point | |
| **Total:** | **4 points** | |

| **15. c)** | | |
|---|---|---|
| 5 participants will receive financial support. | 1 point | |
| The number of all cases is $\binom{25}{5}$, | 1 point | |
| The number of favourable cases is $\binom{17}{5}$. | 1 point | |
| (According to the classical model, the probability is) $\frac{\binom{17}{5}}{\binom{25}{5}} = \frac{6188}{53130} \approx 0.1165.$ | 1 point | |
| The probability of selecting 5 women is 0.12 (that is, 11.65%). | 1 point | |
| **Total:** | **5 points** | |

# II. B

| 16. a) | | |
|---|---|---|
| If $c$ is the length of the third side of the triangle, (it follows from the triangle inequality that) $20+c>22$ | 1 point | |
| and $c<20+22$. | 1 point | |
| Therefore $2<c<42$. | 1 point | |
| If the third side is also an integer then the smallest possible value of $c$ is 3, and its largest possible value is 41. | 1 point | |
| That means 39 possible triangles. | 1 point | |
| **Total:** | **5 points** | |

| 16. b) | | |
|---|---|---|
| (If γ denotes the angle enclosed by the two given sides,) $88=\frac{20\cdot 22\cdot \sin\gamma}{2}$. | 1 point | |
| Hence $\sin\gamma=0.4$. | 1 point | |
| $\gamma_1\approx 23.6^\circ$ | 1 point | |
| $\gamma_2\approx 156.4^\circ$ | 1 point | |
| **Total:** | **4 points** | |

| 16. c) | | |
|---|---|---|
| For $\gamma_1\approx 23.6^\circ$, the length ($c_1$) of the third side is obtained by using the cosine rule. | 1 point | *The point is also due if this idea is only reflected by the solution* |
| $c_1^2\approx 20^2+22^2-2\cdot 20\cdot 22\cdot\cos 23.6^\circ$ | 1 point | |
| $c_1^2\approx 77.568$. | 1 point | |
| Hence $c_1\approx 8.8$ units long. | 1 point | |
| For $\gamma_2\approx 156.4^\circ$ and the length ($c_2$) of the third side: $c_2^2\approx 20^2+22^2-2\cdot 20\cdot 22\cdot\cos 156.4^\circ$. | 1 point | |
| $c_2^2\approx 884-880\cdot(-0.9164)$, that is $c_2^2\approx 1690.4$ and hence | 1 point | |
| $c_2\approx 41.1$ units long. | 1 point | |
| The length of the third side of the triangle may be $\approx 8.8$ units or $\approx 41.1$ units long. | 1 point | |
| **Total:** | **8 points** | *Award 4 points at most for this part if only one of the two cases is investigated.* |

| 17. a) | | |
|---|---|---|
| The rent to be paid by Gábor increases as a geometric progression: $a_1 = 100$ and $a_{24} = 200$. | 1 point | |
| $100 \cdot q^{23} = 200$, $q^{23} = 2$ (where the common ratio is $q = 1 + \frac{p}{100}$) | 1 point | |
| $q = \sqrt[23]{2} = 2^{\frac{1}{23}}$ $(\approx 1.0306)$ | 1 point | |
| $p = 3.06$, | 1 point | |
| that is, Gábor pays 3.06% more each month. | 1 point | |
| **Total:** | **5 points** | |

| 17. b) | | |
|---|---|---|
| Péter's rent follows an arithmetic progression: $b_1 = 100$ and $b_{24} = 200$, | 1 point | |
| $200 = 100 + 23 \cdot d$ | 1 point | |
| $d = \frac{100}{23} \approx 4.35$, that is, the monthly increment is 4.35 dollars. | 1 point | |
| **Total:** | **3 points** | |

| 17. c) | | |
|---|---|---|
| The sums of the first 24 terms of the individual sequences: | 1 point | *The point is also due if this idea is only reflected by the solution.* |
| $S_{Gábor} = 100 \cdot \frac{\left(\sqrt[23]{2}\right)^{24} - 1}{\sqrt[23]{2} - 1} \approx 3468.45$. | 2 points | |
| $S_{Péter} = \frac{100 + 200}{2} \cdot 24 = 3600$. | 2 points | |
| Péter pays 132 dollars more rent than Gábor during the 24 months. | 1 point | |
| **Total:** | **6 points** | |
| *If the monthly rents are represented graphically, it shows that Péter would pay more rent than Gábor in each month (except the 1st and 24th months) Therefore Péter would clearly pay more during the 24 months than Gábor. 3 points may be awarded for a reasoning based on clear graphs.* | | |

| **17. d)** | | |
|---|---|---|
| During the first 12 months, Péter pays a total rent of $S_{12}=\frac{2\cdot 100+11\cdot\frac{100}{23}}{2}\cdot 12\approx 1487$ dollars, | 1 point | |
| and he pays 2113 dollars in the second 12 months. | 1 point | |
| $\frac{2113}{1487}\approx 1.421$, thus Péter pays 42.1% more rent in the second year than in the first year. | 1 point | |
| **Total:** | **3 points** | |

| **18. a) Solution 1** | | |
|---|---|---|
| If we disregard the restriction that the two products cannot pe placed next to each other then the number of orders is 6! | 1 point | |
| If the two products are placed next to each other but their order is not considered, the number of possible arrangements is 5!. | 1 point | |
| If the order of the two product is also considered, there are $2\cdot 5!$ arrangements of the six products. | 1 point | |
| The number of orders in question is obtained by subtracting these from the number of all cases: $6!-2\cdot 5!$. | 2 points | *The 2 points are also due for a reasoning less detailed.* |
| Thus the number of possible orders of the six products is 480. | 1 point | |
| **Total:** | **6 points** | *Award at most 1 point if the restriction on two kinds of product not being next to each other is disregarded.* |

| **18. a) Solution 2** | | |
|---|---|---|
| Semolina and bread crumbs alone can be placed on the shelf in $6\cdot 5$, that is, 30 ways if they are allowed to be next to each other as well. | 1 point | |
| If their order is not considered, they may be next to each other in 5 places. | 1 point | |
| Since order also matters, that makes 10 cases. | 1 point | |
| Thus there are $(30-10=)$ 20 ways to place these two product on the shelf so that they are not next to each other. | 1 point | |
| In each of the 20 cases, the remaining four products may be arranged in 4! different ways. | 1 point | |
| Therefore the total number of possible orders is $20\cdot 4!=480$. | 1 point | |
| **Total:** | **6 points** | |

| **18. b)** | | |
|---|---|---|
| The number of bread loaves was 325 (=176+109+40), and 42 were sent back. | 1 point | |
| This is 12.9% of the quantity ordered. | 1 point | |
| The total number of rolls and croissants was 695 (=314+381), and 34 were sent back. | 1 point | |
| This is 4.9% of the quantity ordered. | 1 point | |
| **Total:** | **4 points** | |

| **18. c)** | | |
|---|---|---|
| The numbers of rolls and croissants together, sold on the individual days are 124; 133; 132; 122; 150. | 1 point | |
| The two days can be selected in $\binom{5}{2}$ ways. | 1 point | |
| (There were 3 days when at least 130 pieces were sold.) The two days can be selected in $\binom{3}{2}$ ways, | 1 point | |
| Thus the probability in question is $\frac{\binom{3}{2}}{\binom{5}{2}} = 0.3$ | 1 point | |
| **Total:** | **4 points** | |

| **18. d)** | | |
|---|---|---|
| The quantities ordered were: 1-kg loaves of white bread: ($\frac{155}{5}=$) 31, 1/2-kg loaves of white bread: ($\frac{95}{5}=$) 19, loaves of rye bread: ($\frac{33}{5}=6.6$) 7, | 2 points | *1 point for two correct answers, no points for only one correct answer.* |
| bread rolls: 58, croissants: 74. | 1 point | |
| **Total:** | **3 points** | |